## FUNDAMENTAL THEORY OF HEAT PIPES

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Among the problems involved in the theory of heat pipes is that of finding the maximum amount of heat which can be transferred along the pipe, the closely related problem of finding the optimum design for heat pipes, and the problem of determining the thermal resistance of pipes. Successful and efficient applications of heat pipes depend on the solution of these problems. It has generally been believed that the thermal resistance of heat pipes is negligible, except in cases in which the compressibility of the vapor comes into play, so that most work on this theory has dealt with only the first two of these problems. Furthermore, despite the large number of papers which have been published, the theory which has been worked out is generally approximate or of a qualitative-estimate nature, and very restrictive assumptions have been used. The problem of designing heat pipes has thus been neither worked out fully nor formulated clearly. It is sufficient to note that all published work on the design of heat pipes has been restricted to either long pipes or to conditions on the outer surface of the pipe such that the heat flow through the pipe is constant in the evaporation and condensation zones.

Recent experiments have shown that under certain working conditions heat pipes can display a significant thermal resistance which cannot be attributed to vapor compressibility. In particular, this is the implication of measurements of the vapor temperature inside a sodium heat pipe [1].

To derive a theory giving the thermal resistance of a pipe which holds for short pipes and which takes into account the heat-transfer conditions (which are highly inhomogeneous in the longitudinal direction) on the outer surface of the pipe, we must start from a rigorous formulation of the problem of designing heat pipes, taking into account all the basic heat- and mass-transfer processes in the pipe material and in the cavity simultaneously; i.e., we must formulate the problem as a conjugate problem in the sense of [2].

1. One of the initial considerations for determining the limiting capabilities of a heat pipe is the set of working conditions. Various versions of these conditions are given in [3, 4], but all are only qualitative in nature, as can be seen from the circumstance that these conditions, representing a limitation on the total pressure drop in the pipe, do not specify between which points on the interface this pressure drop is to be determined.

A rigorous working condition, free of this shortcoming, was found in [5]:

$$\max_{\substack{0 \le x_1 \le l \\ 0 \le x_2 \le l}} [(P_{V1} - P_{V2}) - (P_{L1} - P_{L2}) - (\rho_{V1} v_{c1}^2 - \rho_L u_{c1}^2) - (\rho_{V2} v_{c2}^2 - \rho_L u_{c1}^2)] \le P_{k_{max}}.$$
 (1)

This condition specifies the points on the inner surface of a capillary structure between which the total pressure drop is to be determined; these points are those at which the left side of inequality (1) reaches maxima as a function of the two variables  $x_1$  and  $x_2$ .

The limiting capillary pressure  $P_{k_{max}}$  in condition (1) is governed primarily by the capillary struc-

ture and can be found easily for each particular structure. For example, it is not difficult to see that in the case of a capillary structure having cylindrical pores of radius a the pressure  $P_{k_{max}}$  would be given by

 $P_{k_{\max}} = \frac{2\sigma}{a} \cos \delta - \frac{\sigma}{R} , \qquad (2)$ 

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while in the case of a capillary structure having open channels of width 2a on the inner surface of a heat pipe this pressure would be

$$P_{k_{\max}} = \frac{\sigma}{a} \cos \delta - \frac{\sigma}{R} .$$
(3)

Condition (1) takes into account the reaction of the flows of evaporated and condensed material to the phase-transition surface. This effect turns out to be important only at very high phase-transition velocities and when there is a pronounced conversion of heat flows across the pipe shell in the evaporation and condensation zones. The first attempt to take this effect into account in an analysis of the start-up of a heat pipe was made in [6].

2. A complication which arises in the formulation of the problem of designing heat pipes is that changes in the working conditions can be accompanied by changes in the nature of the transfer processes occurring in the pipe. For example, the vapor flow inside the pipe can be either laminar or turbulent, depending on the heat load and the working temperature, and the vapor itself can be either compressible or incompressible. Transfer processes in the pipe shell can be complicated by boiling of the coolant in the pores of the capillary structure and the partial drying of this structure. It thus becomes necessary to determine the conditions corresponding to each working regime of the heat pipe. A method was described in [5], for example, for determining the heat load and working temperature for which each of these vapor flow regimes occurs.

It should be noted that the structure of the conjugate problem of designing heat pipes is the same for different operating regimes. The only differences are in the individual equations describing those transfer processes which change in nature in the transition to the new working regime. We turn now to the formulation of the conjugate problem of designing heat pipes for the case of the laminar flow of an incompressible vapor [5]. In this case the motion of the vapor is described by

$$\frac{\partial r v_x}{\partial x} - \frac{\partial r v_r}{\partial r} = 0, \tag{4}$$

$$\rho_{\mathbf{V}}v_{x} \frac{\partial v_{x}}{\partial x} + \rho_{\mathbf{V}}v_{r} \frac{\partial v_{x}}{\partial r} = -\frac{\partial P_{\mathbf{V}}}{\partial x} + \eta_{\mathbf{V}} \left( \frac{\partial^{2}v_{x}}{\partial x^{2}} + \frac{\partial^{2}v_{x}}{\partial r^{2}} + \frac{1}{r} \cdot \frac{\partial v_{x}}{\partial r} \right), \tag{5}$$

$$\rho_{\mathbf{V}} v_{x} \frac{\partial v_{r}}{\partial x} + \rho_{\mathbf{V}} v_{r} \frac{\partial v_{r}}{\partial r} = -\frac{\partial P_{\mathbf{V}}}{\partial r} - \eta_{\mathbf{V}} \left( \frac{\partial^{2} v_{r}}{\partial x^{2}} + \frac{\partial^{2} v_{r}}{\partial r^{2}} - \frac{1}{r} \cdot \frac{\partial v_{r}}{\partial r} - \frac{v_{r}}{r^{2}} \right), \tag{6}$$

$$\rho_{\mathbf{V}}c_{\rho_{\mathbf{V}}}v_{x}\frac{\partial T_{\mathbf{V}}}{\partial x} - \rho_{\mathbf{V}}c_{\rho_{\mathbf{V}}}v_{r}\frac{\partial T_{\mathbf{V}}}{\partial r} = \lambda_{\mathbf{V}}\left(\frac{\partial^{2}T_{\mathbf{V}}}{\partial x^{2}} + \frac{\partial^{2}T_{\mathbf{V}}}{\partial r^{2}} + \frac{1}{r}\cdot\frac{\partial T_{\mathbf{V}}}{\partial r}\right).$$
(7)

If the thermal conductivities of the coolant and the material of the capillary structure are approximately the same, as they are in high-temperature heat pipes, the coolant temperature must be at approximately the same temperature as the capillary structure, so that the energy transport in a capillary structure whose pores are completely filled with condensate can be described by

$$\frac{\partial}{\partial x} \left[ r \left( \tau_{\mathbf{L}_{x}} h_{\mathbf{L}} - \dot{\lambda}_{x}' \frac{\partial T}{\partial x} \right) \right] + \frac{\partial}{\partial r} \left[ r \left( \tau_{\mathbf{L}_{r}} h_{\mathbf{L}} - \dot{\lambda}_{r}' \frac{\partial T}{\partial r} \right) \right] = 0.$$
(8)

System (4)-(8) should be supplemented with the heat-conduction equation in the pipe shell and the equation describing the filtering of the coolant through the capillary structure. As a rule, the pipe shell is much thinner than the capillary structure, so that it is frequently possible to neglect the heat redistribution along the pipe shell. Then the heat-conduction equation for the shell can be integrated easily, and we find, in particular, the following relation between the heat flows across the outer and inner surfaces of the pipe shell:  $q_1 = q_2R_2/R_1$ . The particular form of the coolant-filtering equation depends on the type of capillary structure used in the pipe; this aspect of the problem will be discussed a bit further on.

A formulation of the conjugate problem also implies a formulation of the boundary conditions and the conjugate conditions for this system of equations. Let us consider in particular the case in which the conditions at the inner surface of the pipe reduce to a specification of the heat flow across this surface; where necessary, the problem can easily be reformulated for other conditions. We further assume that there is

no heat transfer across the ends of the pipe. Then a steady state is possible only if  $\int_{0}^{1} q_2(x) dx = 0$ .

We introduce the dimensionless quantity  $\epsilon$ , defining it as a quantity which is numerically equal to the surface area at which the phase transition occurs divided by the unit surface area of the capillary structure, r = R. At those parts of the r = R surface where evaporation occurs, the quantity  $\epsilon$  is equal to the surface porosity of the capillary structure,  $\epsilon_{CS}$ . Since vapor condenses over the entire r = R surface, we would have  $\epsilon = 1$  in the condensation zone.

Using this definition we can write the complete set of boundary conditions and conjugate conditions for Eqs. (4)-(8) and for the coolant-filtering equation:

$$v_x|_{x=0} = v_r|_{x=0} = v_x|_{x=l} = v_r|_{x=l} = v_x|_{r=R} = 0;$$
(9)

$$\frac{\partial T_{\mathbf{V}}}{\partial x}\Big|_{x=0} = \frac{\partial T_{\mathbf{V}}}{\partial x}\Big|_{x=1} = 0; \quad \frac{\partial T}{\partial x}\Big|_{x=0} = \frac{\partial T}{\partial x}\Big|_{x=1} = 0; \quad (10)$$

$$-\lambda_r'\frac{\partial T}{\partial r}\Big|_{r=R_1} = q_2 R_2 / R_1;$$
(11)

$$L\tau|_{r=R} = -\lambda_r' \frac{\partial T}{\partial r}\Big|_{r=R} : \quad \tau|_{r=R} = \frac{2\alpha\varepsilon}{2-\alpha} \sqrt{\frac{\mu}{2\pi R^*}} \left[ \frac{P_{\mathbf{V}}}{\sqrt{T_{\mathbf{V}}}} - \frac{P(T_{\mathbf{0}})}{\sqrt{T_{\mathbf{0}}}} \right]_{r=R}; \quad (12)$$

$$T_{\mathbf{V}|_{r=R}} = T_{\mathbf{0}} \left[ 1 + \sqrt{\frac{2\pi R^* T_{\mathbf{0}}}{\mu}} \cdot \frac{\tau}{8\epsilon P(T_{\mathbf{0}}) \left[\alpha - \beta(1-\alpha)\right]} \right]_{r=R} \quad \text{for} \quad \tau|_{r=R} < 0;$$
(13)

$$\frac{\partial T_{\mathbf{V}}}{\partial r}\Big|_{r=R} = 0 \quad \text{for } \tau|_{r=R} > 0;$$
(14)

$$\lambda_{r}^{\prime} \frac{\partial T}{\partial r} \Big|_{r=R} = \lambda_{V} \frac{\partial T_{V}}{\partial r} \Big|_{r=R} \quad \text{for} \quad \tau \Big|_{r=R} = 0.$$
(15)

When the problem is formulated in this manner, there is no characteristic temperature, so that the solution is determined within the value of the temperature at some point on the pipe.

3. To numerically solve this complicated nonlinear problem we must use an iterative process; the most natural procedure would be to numerically simulate the establishment of steady-state operation of the pipe. A necessary but not a sufficient condition for the convergence of this iterative process is spatial stability of the solution found in each step of the iteration. Experience in such calculations has shown that it is extremely difficult to satisfy this condition in the simultaneous solution of the entire system of equations describing the processes in the heat pipe.

However, spatial stability can be achieved if the iteration is based on a successive solution of the problems of the vapor motion inside the pipe, of heat transfer across the pipe shell, and of coolant motion in the capillary structure. Since all these problems are interrelated, the solution of each must be based on all the quantities corresponding to the other problems in the previous step of the iteration. For example, in solving the problem of the vapor motion inside the pipe, we must take the surface temperature  $T_0$  and the heat flow across this surface,  $q_0$ , from the solution of the previous step in the solution of the heat-conduction problem in the pipe shell. The successive solution of these problems, in addition to ensuring spatial stability, makes a considerably lower demand on the operational memory of the computer.

For brevity, we omit discussions of the methods for solving each of these problems, but we do not wish to draw attention to previous comments [5] on differences in the methods for solving the problem of the laminar flow of the vapor in the cases of short and long heat pipes. In [5] there is also a system of equations for approximate calculations of turbulent vapor flow.

The problem of the heat flow through the capillary structure can be solved approximately by specifying the radial dependence of the temperature as some polynomial.

In contrast with the vapor problem, the coolant-filtering problem must take into account the effect of gravity. This can be handled comparatively simply: Since the gravitational force is a potential force, it is sufficient to subtract the quantity  $\rho_L gx \sin \gamma$  from the liquid pressure obtained in a solution of the problem neglecting gravity.

An estimate of Re<sub>rL</sub> for coolant filtering in a heat pipe yields a value less than one for the case of alkali metals, so that we can neglect convective terms in the equations of motion and use the Stokes approximation in calculating the liquid flow through the capillary structure. It can be shown that in this approximation the liquid pressure along the capillary structure can be written



Fig. 1. Cell of the serge network.

$$P_{\rm L}(x) = P_{\rm L}(0) - \frac{v_{\rm L}}{K} d \int_{0}^{x} dx \int_{0}^{x} \tau_{r}|_{r=R} dx, \qquad (16)$$

where K (the permeability) and d are properties of the capillary structure. If the capillary structure is a coaxial gap formed by a small-pore screen and the shell of the heat pipe, these properties are

$$K = \frac{R_1^2}{2} \left[ \frac{1 - \alpha^2}{4} - \frac{1 - \alpha^2}{4 \ln \alpha} \right]; \quad \alpha = \frac{R}{R_1}; \quad d = \frac{2R}{R_1^2 - R^2}.$$
 (17)

The same expression can be used for d if the capillary structure is of an irregular nature (consisting of several layers of a metal grid, a baked powder, etc.).

4. The capillary pressure due to the meniscuses formed in the capillary structure pores depends on the pore radius at the position of a given meniscus. If the capillary structure is such that there are pores of various diameters, it could be that the capillary pressure due to the meniscuses in large pores would not be sufficient to transport the coolant, while the coolant could still move along the small pores. In this case the large pores would dry up either completely or partially, if the condensate does not fill the entire pore cross section. The drying would not occur over the entire length of the capillary, but only beginning at the point at which the difference between the vapor and liquid pressures becomes equal to the limiting surface pressure of the meniscus at the given point. Accordingly, when the working conditions of the heat pipes are such that drying occurs in the capillary structure, the degree to which the capillary structure is filled with coolant varies along the pipe, being complete in the condensation zone but only partial at some point at the beginning of the evaporation zone.

The resistance to the coolant motion, measured by the permeability, depends on this degree of filling, so that the filtering in this case occurs with a permeability which varies along the pipe, and this permeability in turn becomes a function of the filtering process.

When the capillary structure is partially dry, the condensate evaporates in its interior, and the resulting vapor moves along the large pores to the surface of the capillary structure, entering the cavity of the pipe. As the vapor moves along the large pores, the capillary structure is heated as a result of heat transfer. To incorporate these processes in our formulation of the problem we must adopt some model for the capillary structure and for the filtering of the condensate through it under conditions of partial drying.

The assumptions which we adopt regarding the filtering of the vapor and the condensate are that the vapor moves only across the capillary structure in the r direction, while the liquid moves along the capillary structure, in the x direction. Then we would have

$$\tau_{\mathbf{V}x} = \tau_{\mathbf{L}} = 0; \quad \tau_x = \tau_{\mathbf{L}x}; \quad \tau_r = \tau_{\mathbf{V}r};$$
$$q_x = \tau_{\mathbf{L}x}h_{\mathbf{L}} - \lambda'_x \frac{\partial T}{\partial x}; \quad q_r = \tau_{\mathbf{V}r}h_{\mathbf{V}} - \lambda'_r \frac{\partial T}{\partial r}$$

and the continuity and energy equations would become

$$\frac{\partial r \tau_{\mathbf{L}x}}{\partial x} - \frac{\partial r \tau_{\mathbf{V}x}}{\partial r} = 0,$$
(18)

$$\frac{\partial}{\partial x} \left[ r \left( \tau_{\mathbf{V}x} h_{\mathbf{L}} - \lambda_{x}^{'} \frac{\partial T}{\partial x} \right) \right] \div \frac{\partial}{\partial r} \left[ r \left( \tau_{\mathbf{V}r} h_{\mathbf{V}} - \lambda_{r}^{'} \frac{\partial T}{\partial r} \right) \right] = 0.$$
(19)

As before, the thermal conductivity coefficients of the coolant and the capillary material are assumed to be approximately equal and quite high, so that a single temperature can be used for both materials. Taking into account the physical meaning of each of the terms in Eq. (19), we can rewrite this equation as

$$\tau_{\mathbf{L}x} \frac{\partial h_{\mathbf{L}}}{\partial x} - \frac{\partial}{\partial x} \left( \lambda'_{x} \frac{\partial T}{\partial x} \right) = -\left( h_{\mathbf{V}}^{\text{evap}} - h_{\mathbf{L}} \right) \frac{1}{r} \cdot \frac{\partial r \tau_{\mathbf{V}r}}{\partial r} + \frac{1}{r} \cdot \frac{\partial}{\partial r} \left( r \lambda'_{r} \frac{\partial T}{\partial r} \right) - \tau_{\mathbf{V}r} \frac{\partial h_{\mathbf{V}}}{\partial r} , \qquad (20)$$

where the enthalpy of the vapor resulting from the evaporation,  $h_{V}^{evap}$  is evaluated at the temperature

$$T_{\mathbf{V}}^{\text{evap}} = T + \frac{T^{3/2}}{4P(T)} \cdot \frac{\alpha}{(2-\alpha)\left[\alpha + \beta\left(1-\alpha\right)\right]} \left[\frac{P_{\mathbf{V}}}{VT_{\mathbf{V}}} - \frac{P(T)}{\sqrt{T}}\right].$$
(21)



Fig. 2. Limiting heat load (watts per square meter) as a function of the length of the pipe (meters) for a Na coolant and  $R_1 = 1 \text{ cm.} 1$ ) 1175°K,  $\gamma = 0$ ; 2) 1075°K,  $\varphi = 0$ ; 3) 973°K,  $\gamma = 0$ ; 4) 973°K,  $\gamma = \pi/2$ ; 5) 973°K,  $\gamma = 0$ .

Fig. 3. Optimum radius of the perforated screen as a function of the pipe length (see Fig. 2 for the meaning of the curve labels).

The variation in the vapor flow along the r direction, reflected in Eq. (20) by the expression  $\partial \tau_{nr}/\partial r$ , is related to the evaporation within the capillary structure. If we denote by  $\varepsilon_i$  the total free surface area of the liquid in a unit volume of the capillary structure, then by using Eq. (12) for the evaporation rate we can write

$$\frac{\partial r\tau_{\mathbf{V}_{\mathbf{r}}}}{\partial r} = -\frac{2\alpha}{2-\alpha} \sqrt{\frac{\mu}{2\pi R^*}} \left[ \frac{P_{\mathbf{V}}}{1 \overline{T}_{\mathbf{V}}} - \frac{P(T)}{1 \overline{T}} \right] r\varepsilon_1.$$
(22)

We take the heat transfer between the vapor and the capillary structure into account by means of the appropriate heat-transfer coefficient. Then introducing the porosity of the dry pores,  $\varepsilon_r^{dry}$ , for a cross section of the capillary structure normal to the r direction, we find

$$\tau_{\mathbf{V}}, \frac{\partial h_{\mathbf{V}}}{\partial r} = \frac{2\lambda_{\mathbf{V}} \operatorname{Nu} f_{1}}{\alpha^{2}} \left(T - T_{\mathbf{V}}\right) \varepsilon_{r}^{\operatorname{dry}}.$$
(23)

Equations (20)-(23) describe the heat flow through the partially dry parts of the capillary structure. Where this structure is completely filled with coolant, the heat flow is described by Eq. (8).

To determine the liquid pressure  $P_L$  and the quantities  $\varepsilon_1$  and  $\varepsilon_r^{dry}$  which appear in (22) and (23), we turn to the example of the capillary structure used by Ivanovskii et al., [1], consisting of several layers of a serge network. During drying, the liquid moves in this material along channels of wedge-shaped cross section formed by the wire of the base and the wires of the weft, which are pressed tightly against the base [1]; i.e., in a capillary structure of this type the number of channels through which the liquid moves remains constant as drying occurs, and only the cross section filled by coolant varies. In the case of partial drying this capillary structure is thus equivalent to a series of capillaries arranged along the pipe. It can be shown that the permeability of a series of capillaries is related to the average cross-sectional area of the liquid-filled capillaries, s, and to the density of these capillaries, n, by

$$K = f s^2 n,$$

where f is a coefficient governed by the geometry of the capillary: the way it twists and its cross-sectional shape.

From Fig. 1, we find the approximate equation  $s = 2r_k^2 \sqrt{R_{ini}/r_k}$  for the cross-sectional area of the wedge-shaped channel; then the permeability of the capillary structure is

$$K(x) = \begin{cases} f \cdot 4r_{\mathbf{k}}^{3}R_{\mathbf{ini}}n, & 0 < x < x_{\mathbf{dry}}, \\ K_{\mathbf{cs}}, & x_{\mathbf{dry}} < x < l, \end{cases}$$
(24)



Fig. 4. Profiles of the vapor pressure (newtons per square meter) and the liquid pressure along the pipe, for the case of a Na coolant,  $R_1 = 1$  cm, and 973°K. Upper pair of curves) l = 1 cm;  $\gamma = 0$ ; middle) l = 0.5 m,  $\gamma = \pi/2$ ; lower) l = 1 m,  $\gamma = \pi/2$ .

where  $K_{CS}$  is the permeability of a capillary structure completely filled with liquid, and x is the point at which drying begins, found from

$$(P_{\mathbf{V}} - P_{\mathbf{L}})_{x=x \, \mathrm{dry}} = \frac{2\sigma \cos \delta}{a} \,. \tag{25}$$

The channel half-width rk is governed by the pressure difference across the free surface of the liquid:

$$r_{\rm k} = \frac{\sigma \cos \delta}{P_{\rm V} - P_{\rm L}}$$
(26)

We note that Eqs. (25) and (26) neglect the (usually inconsequential) reaction at the phase transitions [see (11)]. Then the coolant filtering is described by the Darcy equation

$$\frac{dP_{\rm L}}{dx} = -\frac{v_{\rm L}}{K} \tau_{\rm Lx},\tag{27}$$

where the permeability of the capillary structure is given by (24) and (26). To use (24)-(27) to determine the liquid pressure along the pipe we must know the value of this pressure at some point in the capillary structure. During drying, an excess of coolant appears and is displaced toward the condensation zone by the vapor flow. Accordingly, at the point of minimum curvature of the free surface of the liquid  $x_{min}$ , this surface coincides with the surface of the pipe cavity, so that we have

$$P_{\rm L} \left|_{x_{\rm min}} = P_{\rm V} \right|_{x_{\rm min}} - \frac{\sigma}{R} \,. \tag{28}$$

where x<sub>min</sub> is found from

$$\frac{d}{dx} \left[ P_{\mathbf{V}} - P_{\mathbf{L}} \right]_{x_{\min}} = 0, \tag{29}$$

and where  $dP_{kk}/dx$  is given by (27). If Eq. (29) has no root in the condensation zone, we would have  $x_{min} = l$ .

On the basis of the discussion above regarding the coolant filtering and on the basis of the structure of a capillary structure made from a serge network, we can set  $\varepsilon_1 = 2r_kn$ , and set  $\varepsilon_r^{dry}$  equal to the porosity of the capillary structure  $\varepsilon_{csr}$ . We can also expect the thermal conductivity coefficients  $\lambda_x^i$  and  $\lambda_r^i$  to be given approximately by

$$\lambda'_{x} = \lambda_{\text{solid}} (1 - \varepsilon_{\text{cs}x}); \ \lambda'_{r} = \lambda_{\text{solid}} (1 - \varepsilon_{\text{cs}r}).$$

5. To explain the vapor-temperature drops observed along the pipe in [1], the calculation procedure was completed and converted into a computer program. To facilitate a comparison with the data of [1], the calculations were carried out for the case of a long heat pipe with a sodium coolant, working under conditions of turbulent vapor flow. It was not possible to determine whether there was partial drying of the

capillary structure from the data given in [1], so calculations were carried out for the cases with and without drying. The results show that the significant changes observed in the vapor temperature along the evaporation zone were due to a partial drying of the capillary structure.

Using condition (1), we also carried out calculations designed to find the optimum geometric parameters of heat pipes and to determine their limiting capabilities.

The left side of inequality (1) (which we denote by  $\Delta P_{max}$ ) depends on the geometric parameters of the heat pipes (l,  $l_1$ ,  $l_2$ , R, R<sub>1</sub>); the heat flow along the pipe, Q; the working temperature; and the properties of the capillary structure and the coolant. As Q increases,  $\Delta P_{max}$  increases monotonically, so that the heat load for which (1) is an equality is the limiting heat load for the given pipe. Obviously, for a given value of R<sub>1</sub> and for otherwise equal conditions, there is an optimum radius R of the pipe cavity, at which the capillary structure must overcome the minimum pressure drop. Equating this pressure drop to the limiting capillary pressure P<sub>kmax</sub>, we find the following relation for the maximum heat flow Q<sub>max</sub> which can be maintained along the pipe:

$$\min_{0 < R < R} \left[ \Delta P_{\max} \left( Q_{\max} \right) \right] = P_{k_{\max}}.$$

It should be noted that condition (30) is a restriction on the value of Q which is set by the capabilities of the capillary structure; the actual value of  $Q_{max}$  may turn out to be considerably lower, because of the limitation on the heat flux density through the pipe shell in the evaporation zone imposed by boiling of the coolant and the consequent disruption of operation.

We turn now to some results calculated in an effort to use Eq. (30) to optimize the operation of sodium heat pipes having a capillary structure consisting of a coaxial gap formed by the pipe shell and a perforated screen. Preliminary estimates showed that in all cases considered the reaction at the phase transitions, reflected by the last two expressions in parentheses in  $\Delta P_{max}$  is very insignificant, so these expressions were neglected in the subsequent calculations.

Calculations were carried out for a radius of 0.1 mm for the screen apertures, a screen porosity of 0.5, and condensation and accommodation coefficients of  $\alpha = 0.1$  and  $\beta = 0.1$ . Figure 2 (curves 1, 2, and 3) show, for three temperatures, the limiting heat flux density along the pipe as a function of the length of the pipe for proportionate changes in each zone  $(l_1/l = 0.36; l_2/l = 0.5; R_1 = 1 \text{ cm})$ . The value of  $Q_{\text{max}}$  for a vertical pipe (curve 4) is just a bit higher than for the horizontal case. For values of l slightly larger than 1 m, the coolant cannot be held in the gap at all at T = 973 K because of gravity, and the gap dries up. In this case the coolant does not return to the evaporation zone by means of the capillary structure; instead, as in pipes without capillary structures, it returns through the drainage of a film of condensate along the inner surface of the pipe. It should be noted that if there were no difference between the vapor pressures in the evaporation and condensation zones opposing the force of gravity the coaxial gap would dry up even at l = 0.5 m. The heat pipe considered here, having a screen-aperture radius a = 0.1 mm and a length of no more than 0.5 m, cannot work against gravity. Also shown in this figure (curve 5) is the dependence of  $Q_{\text{max}}$  on l for changes in the length of only the adiabatic zone (the lengths of the evaporation and condensate). In this case  $Q_{\text{max}}$  falls off much more rapidly with increasing l than in the case of proportionate changes in each of the zones.

Figure 3 shows the optimum radius of the perforated screen, at which the maximum heat flows  $Q_{max}$  shown in Fig. 2 can be maintained along the pipe. We see that  $R_{opt}$  depends on the length of the pipe, the working temperature, the orientation of the pipe with respect to the force of gravity, and the ratios among the lengths of the variations zones.

Figure 4 shows the profiles of the vapor and liquid pressures along the heat pipe; according to these calculations, the meniscus curvature is minimal at the beginning of the condensation zone,  $x = l_2/l$ , in all three cases. The maximum meniscus curvature, on the other hand, is in the beginning of the evaporation zone,  $x_m = 0$ , in a horizontal pipe, while it is displaced to  $x_m = 0.0833$  for a vertical pipe 0.5 m long and to the end of the condensation zone,  $x_m = 1$  for a vertical pipe 1 m long. These results point out the errors in intuitive arguments about the positions of minimum and maximum meniscus curvature [3, 4] and reemphasize the need to use the heat-pipe working conditions in the form in (1) in calculations.

## NOTATION

x, r are the axes of the cylindrical coordinate system;

P is the pressure;

- is the density; ρ is the edge wetting angle; δ is the surface tension coefficient; σ are the vapor and liquid velocities; v, u is the cavity radius; R are the inner and outer radii of the pipe;  $R_1, R_2$ are the dynamic and kinematic viscosities; η, ν
- λ is the thermal conductivity;
- is the specific heat at constant pressure;
- $_{ au}^{\mathbf{c}\mathbf{p}}$ is the mass flux density;
- is the enthalpy; h
- is the thermal conductivity of coolant-filled capillary structure; λt
- are the condensation and accommodation coefficients; α, β
- P(T)is the saturation vapor pressure at temperature T;
- $T_0$ is the temperature at the r = R surface;
- is the latent heat of vaporization;  $\mathbf{L}$
- is the acceleration due to gravity: g
- is the angle between the pipe and the horizontal; γ
- is the energy flux; q
- is the pipe length: l
- are the coordinates of the beginning and end of the adiabatic zone.  $l_1, l_2$

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